The fatigue behaviour of notched polyethylene as a function of *R*

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The fatigue behaviour of linear polyethylene with a shallow 0.4 mm notch was investigated at 0.5 Hz under plane strain conditions with both square-wave and sine-wave loading. The maximum stress was constant at 10 MPa and R was varied from +1 to −1.5. The rate of damage was monitored by measuring the crack opening displacement, δ. The rate of damage during most of the lifetime could be represented by the equation $d\delta/\delta dN = \beta$ where N is the number of cycles. β exhibited a minimum with respect to R. The time to failure varied inversely with β and exhibited a maximum at $R \approx 0.5$ for a square-wave loading and 0.1 for the sine-wave. Generally, the square-wave produces faster damage than the sine-wave. The results are quantitatively explained in terms of two mechanisms; (1) the time under load, which disentangles the fibrils and (2) the unloading which bends and crunches the fibrils in the damaged zone. A quantitative model was obtained for describing the dependence of β on R.

1. Introduction

Under a constant stress (less than $\sigma_y/2$) polyethylene fails in a brittle manner in the neighbourhood of room temperature by a process of slow crack growth. This process has been extensively studied by Lu and Brown [1, 2, 3] under plane strain conditions and they showed that the rate of damage as measured by the rate of crack opening displacement depended on the stress intensity as

$$\dot{\delta} = AK^n \quad (1)$$

where A depended on molecular weight, temperature, and mode of loading (tension or bending), and n varied slightly only with the mode of loading. Under the same conditions that these fatigue tests were conducted, i.e. plane strain, single edge notched tension specimens, and 42°C, Lu and Brown [1] found that the initial rate of damage was described by

$$\dot{\delta}_0 = 76K^{4.3}\,\mu\text{m min}^{-1}\ (K \text{ in MPa m}^{1/2}) \quad (2)$$

They also found that the time to failure was predicted by

$$t = \frac{\alpha a_0 + \delta_c}{\dot{\delta}_0} \quad (3)$$

where α was the ratio of the thickness of the craze to its length, δ_c was a constant $\approx 20\,\mu$m and a_0 is the depth of the initial sharp notch.

Microscopic observations of the damaged process [1, 2] showed (1) that upon initial loading a craze formed whose dimensions were predicted by the Dugdale theory (2) the craze grew to a critical size δ_c and then the crack began to grow and (3) the crack and craze grew continuously at an increasing rate as K increased.

One purpose of this investigation is to compare the rate of damage under a periodic stress with the rate of damage under a constant stress and also to determine the effect of R. In this investigation the maximum stress was constant and the minimum stress varied so that $R = K_{min}/K_{max}$ varied from −1.5 to 1.0. In many investigations the Paris Equation has been used to describe the crack growth per cycle by

$$da/dN \sim \Delta K^m \quad (4)$$

The Paris Equation has limitations. Hertzberg and Manson [4] reviewed the effect of R for a given value of ΔK and found that for some materials da/dN increased as R increased from 0.1 to 0.5 in other materials da/dn decreased. In this investigation it was found that for a fixed K_{max}, the rate of damage and the time to failure were not monotonic functions of R. The explanation for this behaviour is based on a model of damage that consists of two mechanisms (1) the time under stress which produces disentangling of the fibrils and (2) unloading which bends and crunches the fibrils.

The frequency of 0.5 Hz was chosen because it was low enough to avoid the effect of thermal damage. Both square-wave and sine-wave fatigue loading were used in order to vary the time under stress and the rate of unloading. For all values of R the square-wave produced more damage per cycle. This result agrees with Nishimura and co-workers [5] who found that for $R = -1$ in MDPE resins, fatigue failure was more rapid for a square-wave than for a sine-wave. Hertzberg

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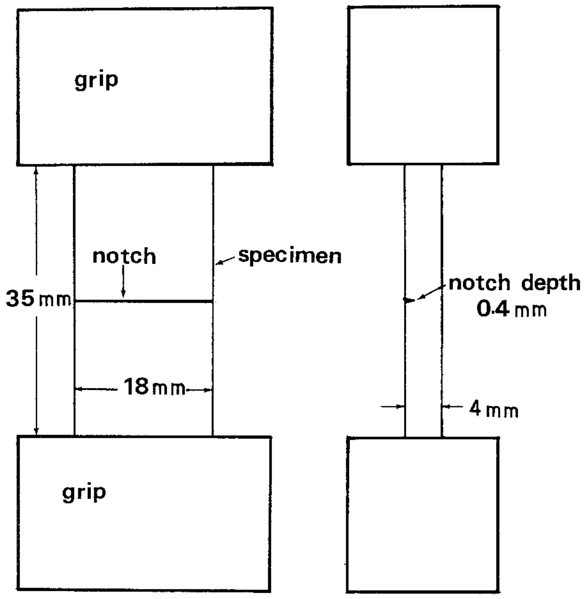

Figure 1 Geometry of specimen.

and Manson [4], in reviewing the effects of wave-form found that the square-wave usually produced failure faster than other wave-forms but it was not always the case.

2. Experimental procedures

The high density polyethylene named Marlex 6006 is made by the Phillips Chemical Company (Bartlesville, Oklahoma, USA) with $M_n = 19\,600$ and $M_w = 130\,000$. The resin was compression moulded into 4 mm thick plaques and very slowly cooled from the melt to a density of 0.964. Its yield point was 25 MPa at 300 K under a strain rate of 0.02 min^{-1}.

The specimen geometry is shown in Fig. 1 where the specimen width was chosen to be 18 mm in order that fracture takes place under essentially plane strain conditions. The notch depth was 0.4 mm so that for most of the life time of the specimen; the crack depth was less than half the thickness of the specimen.

The specimen was gripped in an Instron Model

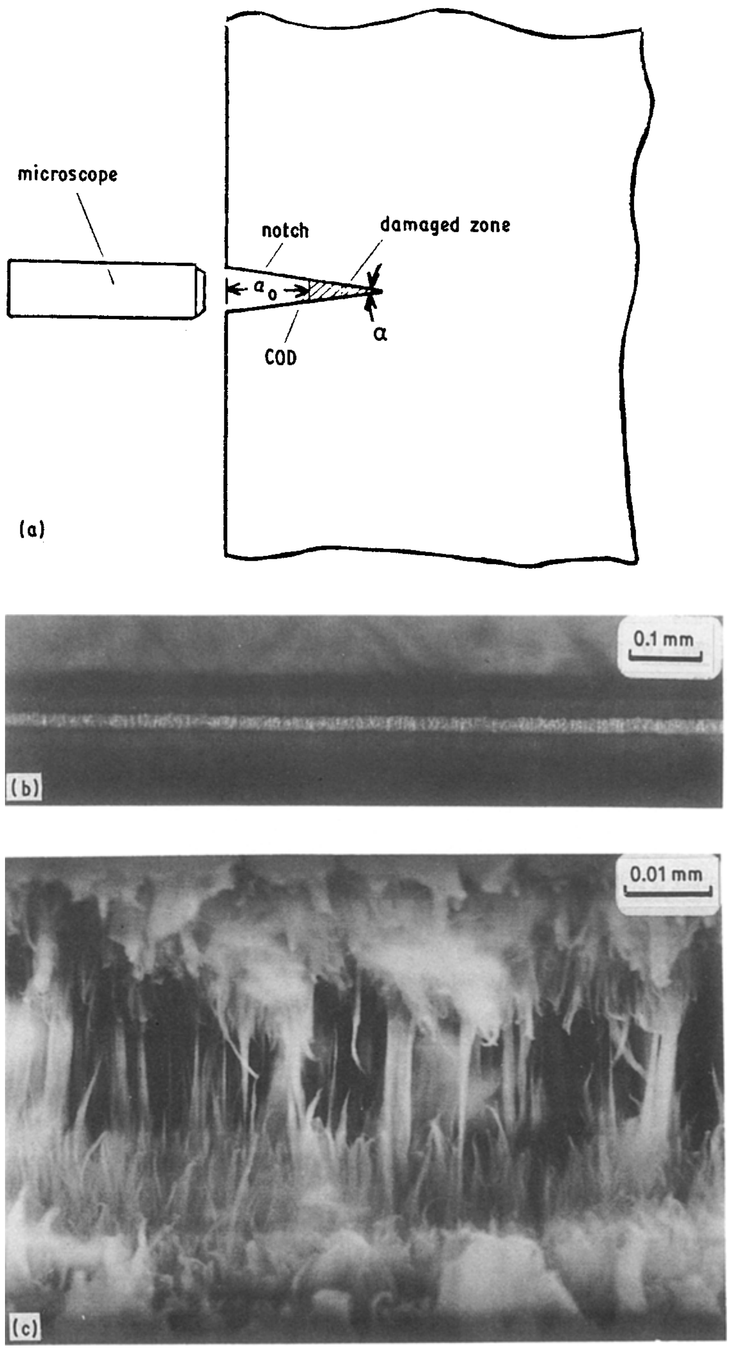

Figure 2 (a) Experimental set up; (b) optical view of notch; (c) SEM view of notch.

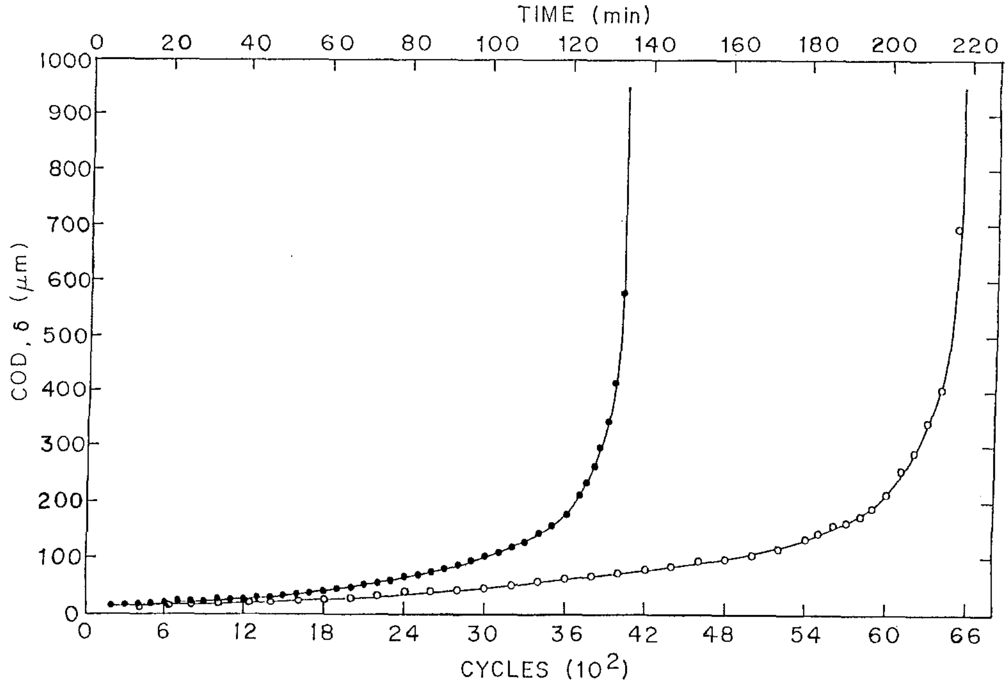

Figure 3 Crack opening displacement, δ, as a function of number of cycles, N, for sine-wave and square-wave loading, 0.5 Hz, 42° C. σ_{max} = 10 MPa. σ_{min} = 1 MPa. (●) square-wave, (O) sine-wave.

1361 electro-mechanical tester. The distance between the grips was 35 mm. The frequency was 0.5 Hz. The temperature was 42 ± 0.2° C. The maximum stress was always 10 MPa and was so chosen because it was less than one-half the yield point. Previous work by Bhattacharya and Brown [6] showed that if the stress is less than one-half the yield point, the zone of damage is essentially planar. The minimum stress varied from − 15 to + 10 MPa. Square-wave and sine-wave loading were investigated.

The damage was monitored by measuring the crack opening displacement at the tip of the original razor blade notch with an optical microscope having a filar eyepiece (Fig. 2). The error in measuring the COD was about ± 2 μm. The damage zone was very bright and clearly visible under the reflected light. Generally, at least two specimens were used at each value of R except in those regions of R where greater scatter required more specimens. The measurement of the COD required about 10 sec and during this time the frequency was reduced to 0.1 Hz.

3. Results

The typical basic experimental observations are shown in Fig. 3 where the crack opening displacement, δ, at the tip of the initial notch is plotted against the number of cycles, N. Figure 3 shows the curves for sine-wave and square-wave loading for the same R. In general, the rate of damage is greater for the square-wave for all values of R that were investigated. Figure 4 shows δ against N, for square-wave loading for R ranging from − 1 to + 1. The effect of R is not readily apparent from Fig. 4. It was decided to plot the rate of damage, $\mathrm{d}\delta/\mathrm{d}N$ as a function of the amount of damage as represented by δ as in Fig. 5. It was found that most of the curve was linear except at the beginning and the end of the fatigue process. This result applied to all values of R. Thus, for the most part, each fatigue curve could be represented by the equation

$$\mathrm{d}\delta/\mathrm{d}N = \beta\delta \qquad (5)$$

This also means that the data can be represented by

$$(N - N_0) = \ln \delta/\delta_0 \, (1/\beta) \qquad (6)$$

where δ_0 is the COD at $N = N_0$.

That plots of $\ln\sigma$ against N are linear is shown in Fig. 6 for various values of R. The general features of the curves in Fig. 6 are as follows: (1) the initial non-linear part of each curve is associated with crack initiation, (2) the linear part is associated with slow crack growth and (3) the final non-linear accelerating part is associated with the yielding and fatigue failure of the remaining ligament. The slope of the linear part of the curves in Fig. 6, β, represents the fractional increase in damage per cycle. In Fig. 7, β is plotted against R where a minimum is observed at $R \approx 0.5$ for square-wave loading.

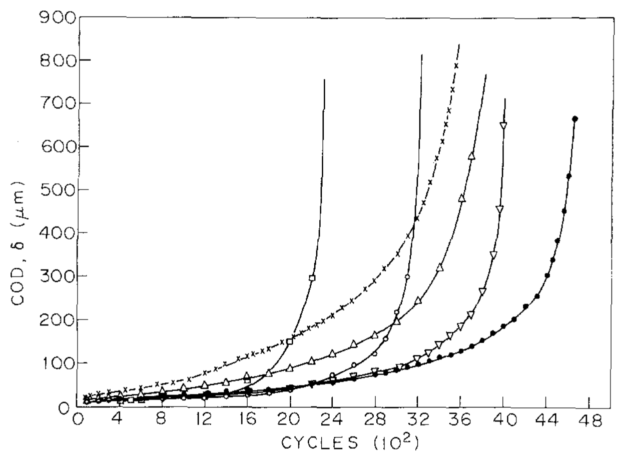

Figure 4 COD, δ, as a function of number of cycles for square-wave and various values of R. σ_{max} = 10 MPa. R values: (□) − 1.0, (O) − 0.5, (x) + 1.0, (△) + 0.7, (▽) + 0.1, (●) + 0.5.

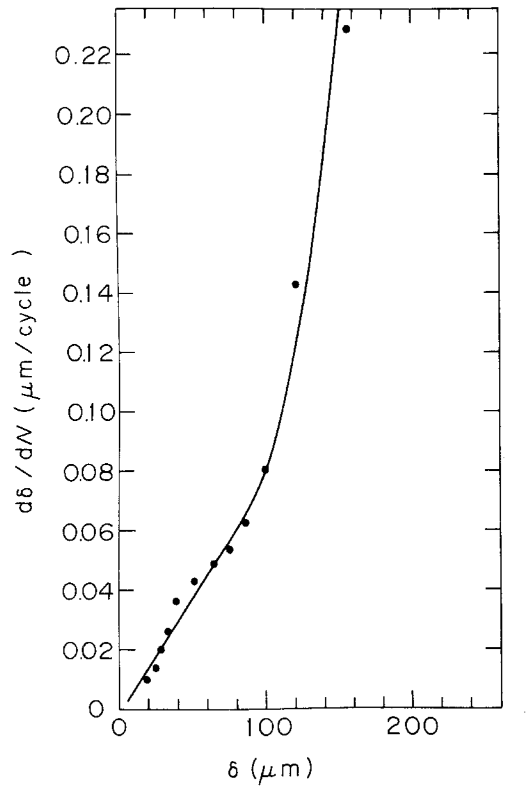

Figure 5 dδ/dN against δ. $R = 0.1$

The cycles for complete failure, N_f, is plotted against R in Fig. 8 where N_f shows a maximum at $R \approx 0.5$ for the square-wave and at $R \approx 0.1$ for the sine-wave. This result is consistent with Fig. 7 since N_f is expected to vary inversely with β. N_f has two parts: (1) the number of cycles during which crack initiation and slow crack growth occur, N_f', and (2) the number of cycles to produce failure after the remaining ligament has yielded, N_y. N_f'/N_y ranged from 3.5 to 7 showing that most of the lifetime involves the slow crack growth process.

The beginning of crack growth was directly observed with the microscope when the fibrils in the zone of damage began to fracture. The number of cycles for crack initiation, N_i, is plotted against R in Fig. 9. N_i shows a maximum at $R = 0.5$ as does N_f, but its maximum is more pronounced than that for N_f.

The microstructures of the damaged zone are shown in Figs 10a, b and c. Fig. 10a shows an early stage of damage prior to crack growth; Fig. 10b shows a damage shortly after crack initiation and Fig. 10c shows a condition when the crack is greater than the fibrillated damage region that precedes it. As shown in Fig. 10b, the leading part of the damage zone consists of an array of micro crazes which ultimately link together to produce crack growth.

Based on the visual observations through the microscope and microphotographs, sketches were made of the damage process as shown in Fig. 11. At the maximum load the fibrils are fully extended. During unloading the fibrils collapse and become bent. At the minimum load for $R < 0.1$ the bent fibrils are crunched. Subsequent reloading to the maximum stress shows that the fibrils have fractured or have become thinner. Damage occurs while the specimen is under the maximum load and also from unloading. Whether more damage is done during unloading or while being crunched under the minimum load for $R < 0.1$ is not known. For $R > 0.1$ crunching of the bent fibrils does not occur. In general, the severity of the bending of the fibrils increases as R decreases.

Microscopic observations of the fractured surface did not reveal any evidence of discontinuous crack growth during the slow crack growth regime. The only discontinuity in the morphology of the fractured surface occurred during the transition from slow crack growth to the yielding of the remaining ligament. The size of the remaining ligament prior to yielding decreased linearly with R as shown in Fig. 12. This

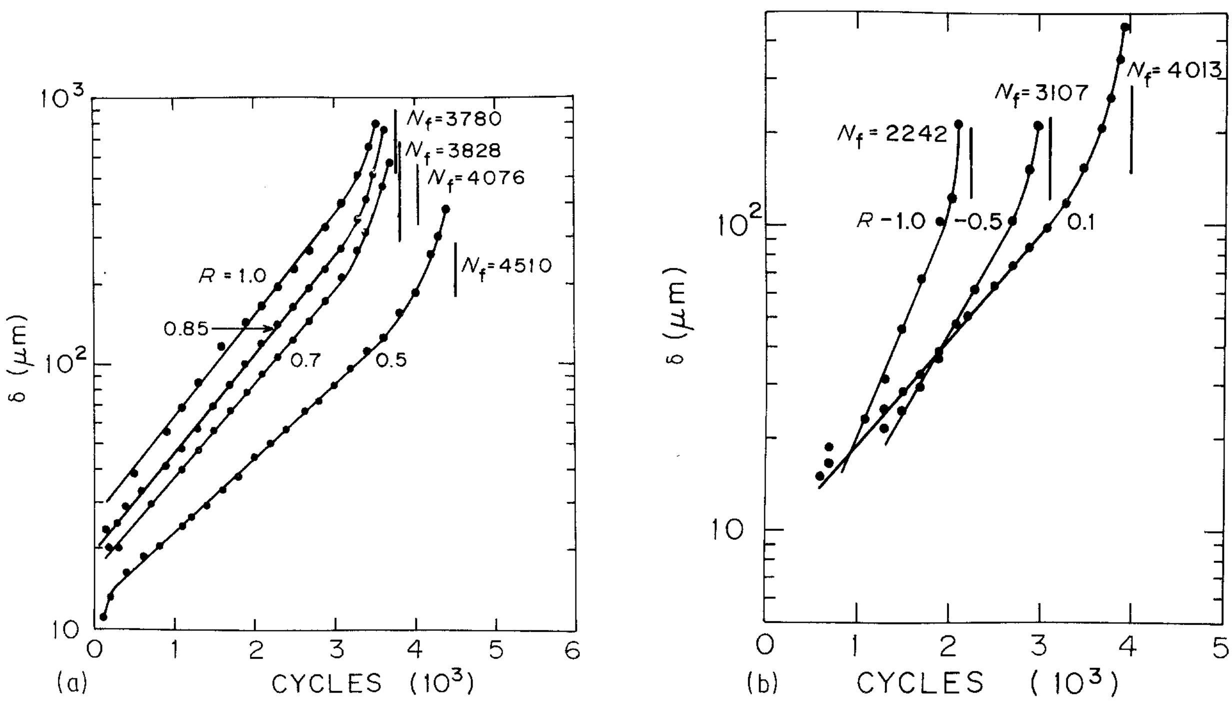

Figure 6 (a) Log δ against N for $R = 0.5, 0.7, 0.85, 1.0$. (b) Log δ against N for $R = -1.0, -0.5, 0.1$.

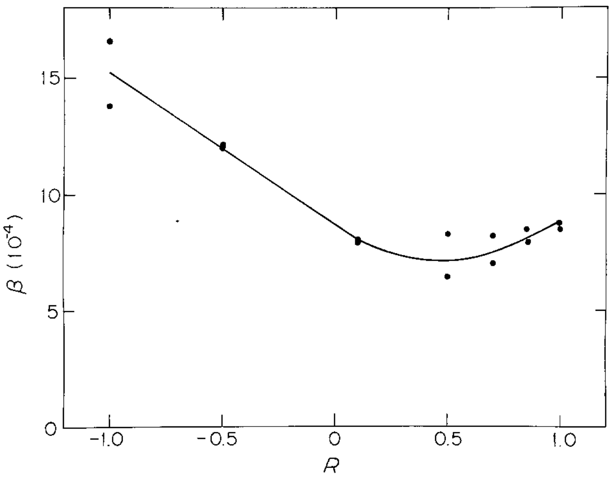

Figure 7 From the slope of the linear part of the curves in Fig. 6, β against R.

result indicates that the yield strength of the remaining ligament increases with R.

4. Discussion

The major goal of this investigation is to determine the effect of R. The interpretation of the results was greatly simplified by the fact that the fatigue damage was observed under plane strain conditions with practically no contribution from shear lips. In the investigation on HDPE by Bucknall and Dumpleton [7] the specimens were 6 mm wide so that fraction of shear lip was large and it also changed as the crack grew. Thus, it is difficult to make a quantitative connection between their results and ours.

The fibrillated region that precedes the crack is generally small compared to the dimensions of the specimen so that fracture mechanics based on small scale yielding is applicable during the period of crack growth. This means that the fatigue processes is dominated by the stress field of the crack.

The geometry of the damage process is illustrated in Fig. 13 where a_0 is the initial notch; Δa is the fractured zone; c is the fibrillated zone; δ is the COD, the observed damage parameter. α is the apex angle of the damage zone and was generally about 10°. Since plane strain prevails over practically all of the damaged

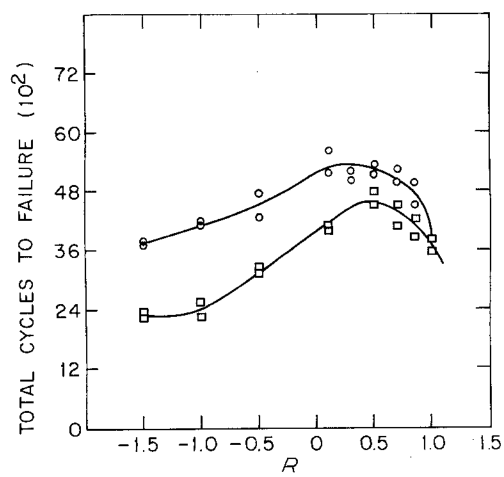

Figure 8 Cycles for complete failure, N_f, against R for sine-wave and square-wave loading. (○) sine-wave, (□) square-wave.

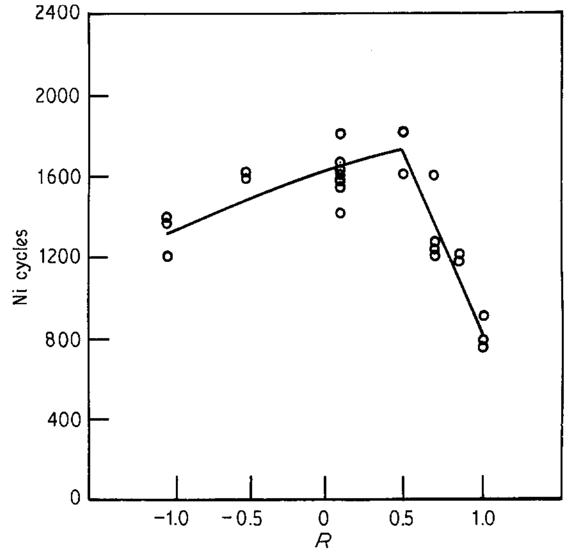

Figure 9 Cycles for the initiation of crack growth as a function of R.

area, Fig. 13 is generally applicable throughout most of the specimen width.

That the slow crack growth process could be described by Equation 4 is a key factor for synthesizing the results. The basic damage process involves the fracture of the fibrils in zone c. When the fibrils fracture, Δa increases and c also increases as predicted by the Dugdale theory. Equation 4 says that the increase in δ per cycle is proportional to δ. We will show that δ is essentially proportional to c so that the damage process which occurs in the fibrillated zone, c, can be more fundamentally described by the equation

$$\mathrm{d}c/\mathrm{d}N = \beta c \tag{7}$$

Equation 7 says that the amount of the fibrillated region that is fractured per cycle is proportional to the size of the fibrillated region. Equation 7 comes from Equation 5 only if δ is proportional to c during crack growth.

From Fig. 13:

$$\delta = \alpha(\Delta a + c) \tag{8}$$

Microscopic measurement of c show that the Dugdale theory gives a good prediction for c. Thus,

$$c = \frac{\pi}{8} K^2/\sigma_y^2 \tag{9}$$

where

$$K = Y\sigma_{\max}(a_0 + \Delta a)^{1/2} \tag{10}$$

and σ_y is the yield point. The values of Y were obtained from Paris and Sih [8].

Combining Equations 8, 9 and 10

$$\frac{\delta}{c} = F\alpha\left[1 - a_0\Big/F\left(\frac{\delta}{\alpha} + a_0\right)\right] \tag{11}$$

$$\text{where } F = \frac{8\sigma_y^2}{\pi Y^2 \sigma_{\max}^2} + 1$$

Since $\sigma_y = 25\,\mathrm{MPa}$; $\sigma_{\max} = 10\,\mathrm{MPa}$ and Y^2 varies from about 2 to 15 as Δa increases. F varies from 2 to 9. $a_0 = 400\,\mu\mathrm{m}$ and $\alpha \approx 10°$. Equation 11 was evaluated for δ values ranging from 10 to 400 μm; it was

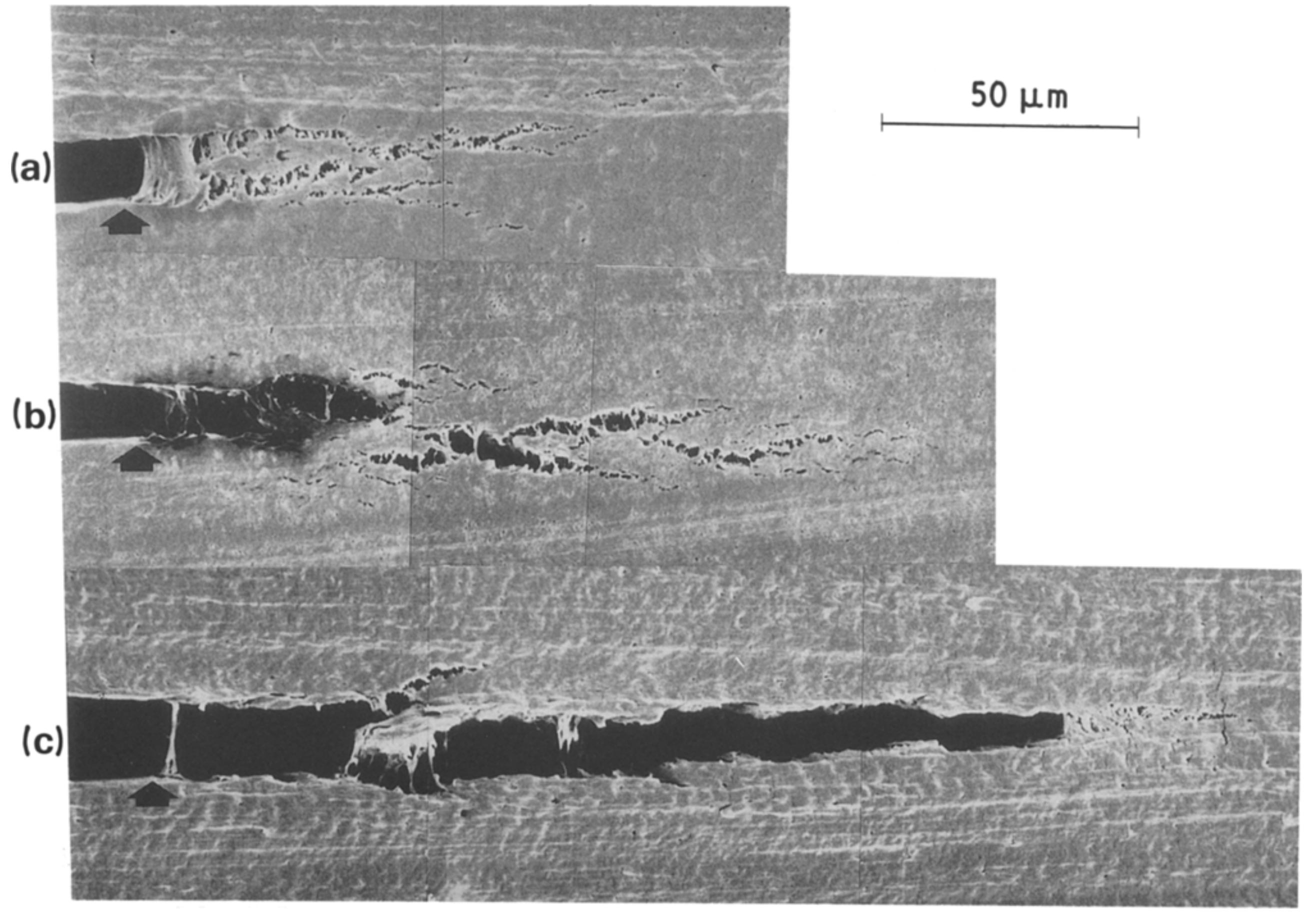

Figure 10 (a) Damaged zone prior to crack growth; (b) Damaged zone shortly after crack growth; (c) Damaged zone when the crack is much larger than the fibrillated region. Arrow indicates root of razor notch where COD is measured.

found that δ/c was practically independent of δ where

$$\delta/c = F\alpha\gamma \qquad (12)$$

where γ varies slightly with δ and F, and for a given value of F changes only about $\pm 10\%$ of its mean value. Since Equation 12 holds, then

$$\Delta\delta/\delta = \Delta c/c$$

where $\Delta\delta$ and Δc are the changes per cycle, and therefore, Equation 5 and Equation 7 are equivalent.

β is taken as the basic measure for the rate of fatigue damage in these experiments. For a given R, β describes the rate of damage during the slow crack growth process. From Fig. 7, β values range from 7 to 15×10^{-4} with an average value of about 10^{-3}. This says that about 0.1% of the fibrillated zone is fractured per cycle. This is the first time that such a simple quantitative measure of the fatigue fracture process has been presented. There are many measurements of the amount of crack growth per cycle using microscopic observations of fatigue striations [4]. Döll measured da/dN as a function of the size of the craze in PMMA and concluded that no simple relationship existed between the size of the craze and da/dN. We have established a simple relationship in the form of Equation 5 or 7 which may only apply to the experimental conditions of this investigation and requires further experiments to determine its generality.

The number of cycles to failure is expected to vary inversely with β as shown by a comparison of Figs 7 and 8. The relationship between cycles to failure and $1/\beta$ comes from Equation 6 where

$$N_f' - N_0 = \ln \delta_f'/\delta_0 \,(1/\beta) \qquad (13)$$

where N_f' is the number of cycles prior to the yield failure of the remaining ligament; δ_f' is the corresponding COD. Fig. 14 is a plot of N_f' against $1/\beta$ for all the experimental data. A linear relationship was obtained.

$$N_f' = 4.6 + 2.8/\beta \;(10^2 \text{ cycles}) \qquad (14)$$

Based on Equation 13, Equation 14 may be interpreted as follows: (1) crack growth initiates after 460 cycles and the COD at failure, δ_f', is 16 times the COD at crack initiation. A survey of all the curves shown in Fig. 6 supports this interpretation. The factor 2.8 in Equation 14 is expected to increase with the thickness of the specimen because the fatigue process was interrupted by the yielding of the remaining ligament. The yielding of the remaining ligament should occur when the applied load divided by the area of the remaining ligament equals the yield point. The area of the remaining ligament is $(b - \delta_f'/\alpha - a_0)t$ where b and t are the width and thickness of the specimen. Therefore,

$$\delta_f' = \alpha\left[b\left(1 - \frac{\sigma}{\sigma_y}\right) - a_0\right] \qquad (15)$$

For this investigation $b = 4\,\text{mm}$, $\sigma = 10\,\text{MPa}$, $\sigma_y = 25\,\text{MPa}$, $a_0 = 0.4\,\text{mm}$ and $\alpha \approx 1/6$. Therefore, $\delta_f' = 330\,\mu\text{m}$ which is in the range of the observed values of δ_f'. However δ_f' increases with σ_y. The fact

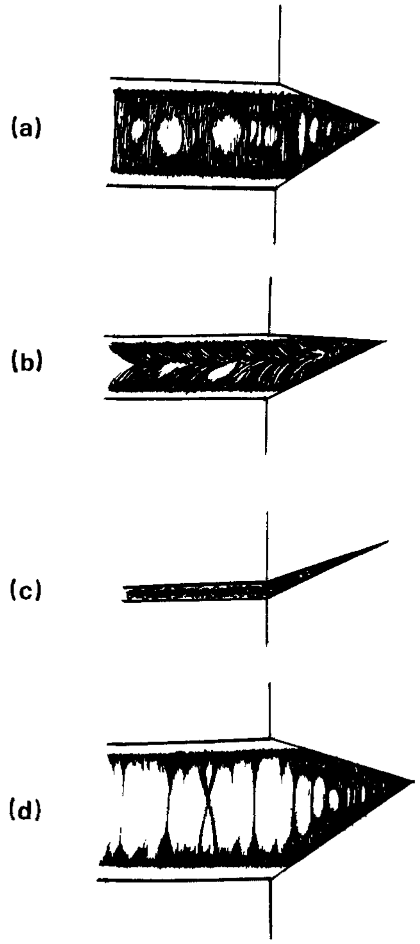

Figure 11 Sequential sketches of fatigue failure based on microscopic observations of side and front views. $R = 0.1$. (a) maximum load; (b) partially unloaded; (c) minimum stress; (d) maximum load after about 50 cycles.

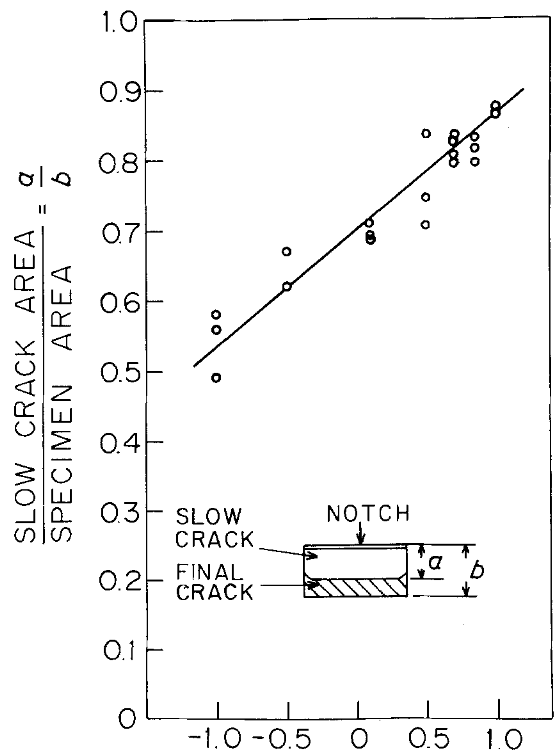

Figure 12 Fraction of the area of the specimen that undergoes slow crack growth prior to the yielding of the remaining ligament as a function of R.

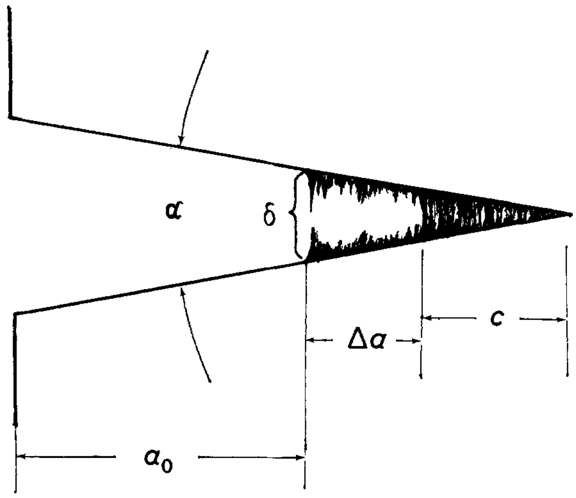

Figure 13 Geometry of the damaged zone.

that the remaining ligament decreases as R increases (Fig. 12) indicates that the yield point of the remaining ligament increases with R. Thus, it is expected that δ_f' should increase with R. An analysis of the curves such as in Fig. 6 show that δ_f' increases from 100 to about 400 μm as R goes from -1 to $+1$. This result suggests that the yield point of the remaining ligament varies from 16 to 25 MPa as R varies from -1 to $+1$. Possibly the lowering of the yield point occurs because alternating the stress between tension and compression tends to soften the polymer whereas a net tension may produce an increase in the yield point by orientation strengthening.

Now the dependence of the rate of damage, β, on R will be explained. Two damage processes occur: (a) β_u, the damage per cycle produced by unloading and (b) β_t the damage that depends on time under load. β_u is associated with the bending and crunching of the fibrils as shown in Figs 11b and c. β_t is associated with

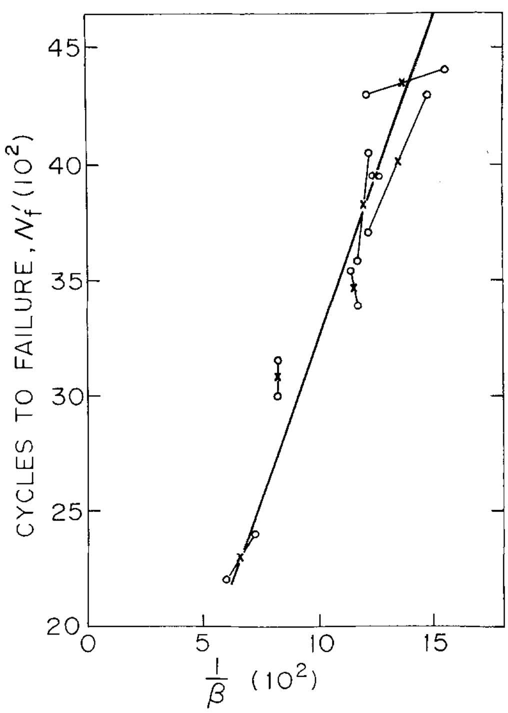

Figure 14 Cycles to failure by slow crack growth, N_f', against $1/\beta$. x = average for two tests with same value of R.

the disentanglement of the fibrils as extensively investigated by Lu and Brown [1-3]. The analysis will be simplified by assuming that $\beta = \beta_u + \beta_t$. No doubt the actual phenomenon is more complex in that there may be a complex interaction between the two damage processes. In accordance with much fatigue data [4], the unloading effect is often described by the Paris Equation

$$\beta_u = \frac{B}{\sigma}(K_{max} - K_{min})^m \qquad (16)$$

where B is a constant.

Lu and Brown [1, 2] have shown that Equation 1 is applicable for the damage rate under constant stress. β_t can be determined from the time dependent damage as follows

$$\beta_t = \frac{d\delta}{\delta dN} = \frac{1}{\delta}\frac{d\delta}{dt(dN/dt)} = \frac{d\delta}{\omega\delta dt} \qquad (17)$$

where ω is the frequency. According to Equations 1 and 5

$$\beta_t = \frac{AK^n}{\omega\delta} \qquad (18)$$

Since δ is proportional to c and $c \sim K^2$ in accordance with the Dugdale theory

$$\beta_u = C(K_{max} - K_{min})^q \qquad (19a)$$

$$\beta_t = \frac{DK^p}{\omega} \qquad (19b)$$

where C and D are constants and $p = n - 2$, $q = m - 2$.

For square-wave loading

$$\beta_t = \frac{D}{2\omega}(K^p_{max} + K^p_{min}) \quad R > 0 \qquad (20a)$$

and

$$\beta_y = \frac{D}{2\omega} K^p_{max} \quad R < 0 \qquad (20b)$$

Combining Equations 19a and 20 where $\beta = \beta_u + \beta_t$

$$\beta = C(1 - R)^q K^q_{max} + \frac{D}{2\omega}(1 + R^p)K^p_{max} \quad R > 0 \qquad (21)$$

$$\beta = C(1 - R)^q K^q_{max} + \frac{D}{2\omega} K^p_{max} \quad R < 0 \qquad (22)$$

$$\frac{(1 - R)^{q-1}}{R^{p-1}} = \frac{pD}{2\omega qC} K^{p-q}_{max} \qquad (23)$$

Equations 21 and 22 can be evaluated from Fig. 7

At $R = 1$, $\beta_u = 0$;

$$\beta_t = \beta = \frac{D}{\omega} K^p_{max} = 8.7 \times 10^{-4}$$

At $R = -1$,

$$\beta_t = 1/2\, \beta(R = 1) = 4.85 \times 10^{-4}$$

and $\beta_u = \beta - \beta_t = (15.2 - 4.85) \times 10^{-4}$
$= 10.85 \times 10^{-4}$

It is also noted from Fig. 7 that β varies linearly with R so that $q = 1$ and

$$2CK_{max} = 10.85 \times 10^{-4}$$

Inserting the above values into Equations 21 and 22

$$\beta = [5.4(1 - R) + 4.4\,(1 + R^p)] \times 10^{-4} \quad R > 0 \qquad (24)$$

$$\beta = [5.4(1 - R) + 4.4] \times 10^{-4} \quad R < 0 \qquad (25)$$

The minimum value of β occurs when

$$R = \left[\frac{5.4}{4.4P}\right]^{1/p-1} \qquad (26)$$

The value of P can be obtained from Lu and Brown [1, 2] where $n = 4.3$ so that $p = 2.3$. Thus, the minimum value of β from Equation 26 is 0.62. The experimental value from Fig. 7 is about 0.5. The value of R at the minimum is not very sensitive to reasonable variations in the choice of p.

When the effect of time under load makes an appreciable contribution to the rate of damage, an equation such as Equations 21 and 22 is superior to the Paris Equation because the maximum stress is taken into account. In the case of sine-wave loading Equation 19 should be modified by using time average values of K^p. Hertzberg and Manson [4] point out that increasing the mean stress may increase or decrease the crack growth rate. This investigation shows that the rate of damage is not a monotonic function of the mean stress when the unloading and time under load effects are both present.

Increasing the maximum stress much beyond $\sigma_y/2$ tends to invalidate Equations 21 and 22 in that the failure mechanism even under a constant load changes. Bhattacharya and Brown [6] showed in contant load experiments that when the applied stress is much greater than one-half the yield point, the deformation zone at the root of the notch changes from a planar to a balloon shape. Consequently, the kinetics of crack growth change. Also, increasing the stress tends to blunt the initial notch so that one may observe a range of stress where the time to failure increases as the stress increases.

Equations 21 and 22 have the following limitations: (1) the maximum stress should be less than about one-half the yield point; (2) the minimum stress cannot become excessively compressive because complex welding effects would occur and (3) the frequency must be sufficiently low that hysteric heating is not significant.

5. Summary

1. For high density PE under plane strain conditions it was observed that the fractional change in the COD per cycle was constant where

$$d\delta/\delta dN = \beta$$

2. β, the parameter for the rate of damage, showed a minimum with respect to R when σ_{max} was held constant.

3. Generally, square wave loading produced damage faster than sine-wave loading.

4. Knowing β, the time to failure can be predicted by Equation 14.

5. The results can be explained in terms of two mechanisms: (1) the effect of unloading and (2) the effect of time under load.

6. A quantitative model for understanding the effect of R on β is given by Equations 21 and 22.

7. The microscopic observations indicate that the damage associated with unloading is caused by bending of fibrils and by the subsequent crunching of the fibrils in the zone of damage that precedes the crack.

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